

120 P28

領域 D は

$$0 \leq x \leq \sqrt{3}, \quad 1 \leq y \leq \sqrt{4-x^2}$$

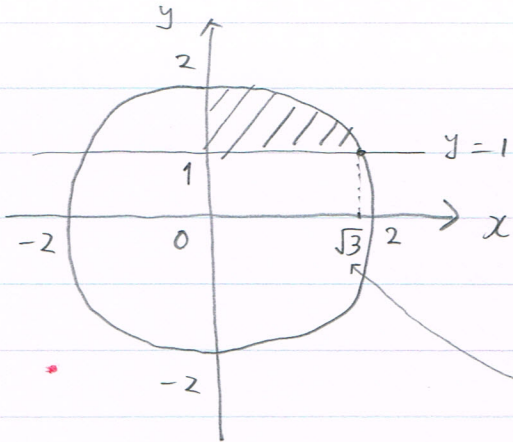
図示する

$$y = \sqrt{4-x^2} \text{ より}$$

$$y^2 = 4-x^2$$

$$x^2+y^2 = 4$$

$$x^2+y^2 = 2^2 \quad (\text{中心原点, 半径 2 の円})$$



$$x^2+y^2 = 2^2 \text{ と } y=1 \text{ の交点は } \pm\sqrt{3}$$

$$x^2+1^2 = 2^2$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$

領域 D は

$$1 \leq y \leq 2, \quad 0 \leq x \leq \sqrt{4-y^2}$$

$$\int_0^{\sqrt{3}} \left\{ \int_1^{\sqrt{4-x^2}} \frac{x}{\sqrt{x^2+y^2}} dy \right\} dx$$

$$= \int_1^2 \left\{ \int_0^{\sqrt{4-y^2}} \frac{x}{\sqrt{x^2+y^2}} dx \right\} dy$$

$$= \int_1^2 \left[ \sqrt{x^2+y^2} \right]_0^{\sqrt{4-y^2}} dy$$

$$= \int_1^2 \left\{ \sqrt{(4-y^2)+y^2} - \sqrt{y^2} \right\} dy$$

$$= \int_1^2 \left\{ \sqrt{4} - \sqrt{y^2} \right\} dy$$

$$= \int_1^2 (2 - |y|) dy$$

←  $1 \leq y \leq 2$  より

$$|y| = y$$

$$= \int_1^2 (2 - y) dy$$

$$= \left[ 2y - \frac{1}{2}y^2 \right]_1^2$$

$$= 2 \cdot 2 - \frac{1}{2} \cdot 4 - \left( 2 - \frac{1}{2} \right)$$

$$= 4 - 2 - 2 + \frac{1}{2}$$

$$= \frac{1}{2}$$

$$x^2+y^2 = 4$$

$$x^2 = 4-y^2$$

$$x = \pm\sqrt{4-y^2}$$

$$x \geq 0 \text{ より}$$

$$x = \sqrt{4-y^2}$$

$$(\sqrt{x})' = \frac{1}{2\sqrt{x}} \text{ より}$$

$$\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + C$$

$$\begin{aligned} \int \frac{x}{\sqrt{x^2+y^2}} dx &= \frac{2x}{2\sqrt{x^2+y^2}} \\ &= \frac{x}{\sqrt{x^2+y^2}} \end{aligned}$$

よって

$$\int \frac{x}{\sqrt{x^2+y^2}} dx = \sqrt{x^2+y^2} + C$$