

□ P73

$$(2) \iint_D \frac{x}{(x+y)^2} dx dy \quad (1 \leq x \leq 2, 0 \leq y \leq 1)$$

$$= \int_1^2 \int_0^1 \frac{x}{(x+y)^2} dy dx$$

$$= \int_1^2 x \int_0^1 \frac{1}{(x+y)^2} dy dx$$

$$= \int_1^2 x \int_0^1 \frac{1}{(y+x)^2} dy dx$$

$$= \int_1^2 x \left[-\frac{1}{y+x} \right]_0^1 dx$$

$$= \int_1^2 x \left\{ -\frac{1}{1+x} + \frac{1}{0+x} \right\} dx$$

$$= \int_1^2 x \left\{ -\frac{1}{x+1} + \frac{1}{x} \right\} dx$$

$$= \int_1^2 \left\{ -\frac{x}{x+1} + 1 \right\} dx$$

$$= -\int_1^2 \frac{x}{x+1} dx + \int_1^2 dx$$

$$= -\int_1^2 \frac{x+1-1}{x+1} dx + [x]_1^2$$

$$= -\int_1^2 \left\{ \frac{x+1}{x+1} - \frac{1}{x+1} \right\} dx + 2 - 1$$

$$= -\int_1^2 \left\{ 1 - \frac{1}{x+1} \right\} dx + 1$$

$$= -\left[x - \log|x+1| \right]_1^2 + 1$$

$$= -\left\{ 2 - \log|3| - (1 - \log|2|) \right\} + 1$$

$$= -\left\{ 2 - \log 3 - 1 + \log 2 \right\} + 1$$

$$= -2 + \log 3 + 1 - \log 2 + 1$$

$$= \log 3 - \log 2$$

$$= \log \frac{3}{2}$$

$$\left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$\left(\frac{1}{x+b}\right)' = -\frac{1}{(x+b)^2}$$

↓

$$\int \frac{1}{(x+b)^2} dx = -\frac{1}{x+b} + C$$

= 5

$$\int_1^2 \frac{x}{x+1} dx \text{ 置換積分でよい}$$

$$t = x+1 \text{ とおく, } x = t-1$$

$$\frac{dt}{dx} = 1 \quad \begin{array}{l} x | 1 \rightarrow 2 \\ t | 2 \rightarrow 3 \end{array}$$

$$dt = dx$$

$$= \int_2^3 \frac{t-1}{t} dt$$

$$= \int_2^3 \left(\frac{t}{t} - \frac{1}{t} \right) dt$$

$$= \int_2^3 \left(1 - \frac{1}{t} \right) dt$$