

③ P135

$$(1) \sin^4 \theta - \cos^4 \theta = 1 - 2\cos^2 \theta$$

〔証明〕

$$(\text{左辺}) = \sin^4 \theta - \cos^4 \theta$$

$$= (\sin^2 \theta + \cos^2 \theta)(\sin^2 \theta - \cos^2 \theta)$$

$$= 1 \times (\sin^2 \theta - \cos^2 \theta)$$

$$= \sin^2 \theta - \cos^2 \theta$$

$$= (1 - \cos^2 \theta) - \cos^2 \theta$$

$$= 1 - 2\cos^2 \theta$$

$$= (\text{右辺})$$

$$\therefore \sin^4 \theta - \cos^4 \theta = 1 - 2\cos^2 \theta$$

$$(2) \tan^2 \theta + (1 - \tan^4 \theta) \cos^2 \theta = 1$$

〔証明〕

$$(\text{左辺}) = \tan^2 \theta + (1 - \tan^4 \theta) \cos^2 \theta$$

$$= \tan^2 \theta + (1 - \tan^2 \theta)(1 + \tan^2 \theta) \cos^2 \theta$$

$$= \tan^2 \theta + (1 - \tan^2 \theta) \cdot \frac{1}{\cos^2 \theta} \cdot \cos^2 \theta$$

$$= \tan^2 \theta + (1 - \tan^2 \theta)$$

$$= 1$$

$$= (\text{右辺})$$

$$\therefore \tan^2 \theta + (1 - \tan^4 \theta) \cos^2 \theta = 1$$

$$\left. \begin{aligned} \sin^2 \theta + \cos^2 \theta &= 1 \\ \sin^2 \theta &= 1 - \cos^2 \theta \end{aligned} \right\}$$

$$\left. \begin{aligned} 1 + \tan^2 \theta &= \frac{1}{\cos^2 \theta} \end{aligned} \right\}$$