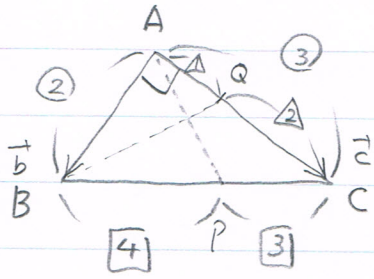


12) 9 p77



$$\vec{AB} = \vec{b}, \vec{AC} = \vec{c} \quad \angle A <$$

$$\vec{AP} = \frac{3\vec{b} + 4\vec{c}}{4+3} = \frac{3}{7}\vec{b} + \frac{4}{7}\vec{c}$$

$$\vec{BQ} = \vec{AQ} - \vec{AB}$$

$$= \frac{1}{3}\vec{c} - \vec{b}$$

$$= -\vec{b} + \frac{1}{3}\vec{c}$$

$$\begin{aligned} \vec{AP} \cdot \vec{BQ} &= \left(\frac{3}{7}\vec{b} + \frac{4}{7}\vec{c}\right) \cdot \left(-\vec{b} + \frac{1}{3}\vec{c}\right) \\ &= -\frac{3}{7}\vec{b} \cdot \vec{b} + \frac{1}{7}\vec{b} \cdot \vec{c} - \frac{4}{7}\vec{b} \cdot \vec{c} + \frac{4}{21}\vec{c} \cdot \vec{c} \\ &= -\frac{3}{7}|\vec{b}|^2 - \frac{3}{7}\vec{b} \cdot \vec{c} + \frac{4}{21}|\vec{c}|^2 \quad \dots \textcircled{1} \end{aligned}$$

$$\because \angle A = 90^\circ \text{ より } \vec{b} \cdot \vec{c} = 0 \quad \dots \textcircled{2}$$

$$\text{また } AB:AC = 2:3 \text{ より } |\vec{b}|:|\vec{c}| = 2:3$$

$$3|\vec{b}| = 2|\vec{c}|$$

$$|\vec{b}| = \frac{2}{3}|\vec{c}| \quad \dots \textcircled{3}$$

$$\textcircled{1} = \textcircled{2}, \textcircled{3} \text{ を代入}$$

$$\begin{aligned} \vec{AP} \cdot \vec{BQ} &= -\frac{3}{7} \cdot \left(\frac{2}{3}|\vec{c}|\right)^2 - \frac{3}{7} \cdot 0 + \frac{4}{21}|\vec{c}|^2 \\ &= -\frac{3}{7} \cdot \frac{4}{9}|\vec{c}|^2 + \frac{4}{21}|\vec{c}|^2 \\ &= -\frac{4}{21}|\vec{c}|^2 + \frac{4}{21}|\vec{c}|^2 \\ &= 0 \end{aligned}$$

よって $AP \perp BQ$