

⑥ P139

$$P(0 \leq x \leq 2) = \int_0^2 f(x) dx = 1 \text{ より}$$

$$\boxed{P} = 1$$

$$\int_0^2 f(x) dx = 1$$

$$\int_0^1 f(x) dx + \int_1^2 f(x) dx = 1$$

$$\int_0^1 ax dx + \int_1^2 a(2-x) dx = 1$$

$$\left[\frac{1}{2}ax^2 \right]_0^1 + \int_1^2 (2a-ax) dx = 1$$

$$\frac{1}{2}a + \left[2ax - \frac{1}{2}ax^2 \right]_1^2 = 1$$

$$\frac{1}{2}a + (4a - 2a) - (2a - \frac{1}{2}a) = 1$$

$$\frac{1}{2}a + 2a - 2a + \frac{1}{2}a = 1$$

$$a = 1 \quad \boxed{1}$$

よって

$$f(x) = \begin{cases} x & (0 \leq x \leq 1) \\ 2-x & (1 \leq x \leq 2) \end{cases}$$

$$P(0.5 \leq x \leq 1.5) = \int_{0.5}^{1.5} f(x) dx$$

$$= \int_{0.5}^1 f(x) dx + \int_1^{1.5} f(x) dx$$

$$= \int_{0.5}^1 x dx + \int_1^{1.5} (2-x) dx$$

$$= \left[\frac{1}{2}x^2 \right]_{0.5}^1 + \left[2x - \frac{1}{2}x^2 \right]_1^{1.5}$$

$$= \frac{1}{2} - \frac{1}{2} \cdot 0.25 + 3 - \frac{1}{2} \cdot 1.5^2 - (2 - \frac{1}{2})$$

$$= \frac{1}{2} - \frac{1}{2} \cdot \frac{25}{100} + 3 - \frac{1}{2} \cdot \left(\frac{3}{2}\right)^2 - 2 + \frac{1}{2}$$

$$= 1 + 1 - \frac{1}{2} \cdot \frac{1}{4} - \frac{1}{2} \cdot \frac{9}{4}$$

$$= 2 - \frac{1}{8} - \frac{9}{8}$$

$$= \frac{16 - 1 - 9}{8}$$

$$= \frac{6}{8} = \frac{3}{4} = \underline{0.75} \quad \leftarrow \boxed{0.75}$$

公式

確率変数 X の確率密度関数が $f(x)$ として
 X の値が $a \leq X \leq b$ となる確率は

- ① $f(x) \geq 0$
- ② $P(a \leq X \leq b) = \int_a^b f(x) dx$
- ③ $P(a \leq X \leq b) = \int_a^b f(x) dx = 1$

公式

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$$