

例22 P30

点 $A(d=c+i)$, $B(B=1)$, $C(r=3i)$

(1) 3点 A, B, C が一直線上にあるとす

$\frac{r-d}{B-d}$ が実数となるので

$$\begin{aligned} \frac{r-d}{B-d} &= \frac{3i-(c+i)}{1-(c+i)} \\ &= \frac{-c+2i}{(1-c)-i} \\ &= \frac{\{-c+2i\}\{(1-c)+i\}}{\{(1-c)-i\}\{(1-c)+i\}} \\ &= \frac{-c(1-c)-ci+2(1-c)i+2i^2}{(1-c)^2-i^2} \\ &= \frac{-c+c^2+\{2(1-c)-c\}i-2}{1-2c+c^2+1} \\ &= \frac{c^2c-2+(2-3c)i}{c^2-2c+2} \\ &= \frac{c^2-c-2}{c^2-2c+2} + \frac{2-3c}{c^2-2c+2}i \end{aligned}$$

実数となるので

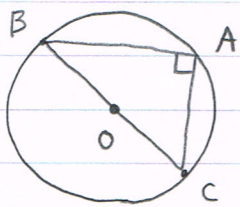
$$\frac{2-3c}{c^2-2c+2} = 0$$

よす

$$2-3c=0$$

$$c = \frac{2}{3}$$

(2)



$\angle BAC = 90^\circ$ となるので

$AB \perp AC$ となる

$\frac{r-d}{B-d}$ が純虚数となる

(1) よす

$$\frac{r-d}{B-d} = \frac{c^2-c-2}{c^2-2c+2} + \frac{2-3c}{c^2-2c+2}i$$

よす

$$\frac{c^2-c-2}{c^2-2c+2} = 0$$

よす

$$c^2-c-2=0$$

$$(c-2)(c+1)=0$$

$$c=2, -1$$

例23 P30

$$2r - (1+\sqrt{3}i)B = (1-\sqrt{3}i)d$$

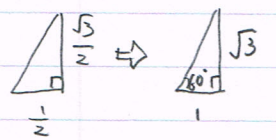
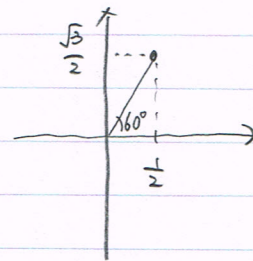
$$\begin{aligned} 2r - (1+\sqrt{3}i)B &= -(1+\sqrt{3}i)d \\ &= -(1+\sqrt{3}i-2)d \\ &= -(1+\sqrt{3}i)d + 2d \end{aligned}$$

$$2r - 2d = (1+\sqrt{3}i)B - (1+\sqrt{3}i)d$$

$$2(r-d) = (1+\sqrt{3}i)(B-d)$$

$$\begin{aligned} \frac{r-d}{B-d} &= \frac{1+\sqrt{3}i}{2} \\ &= \frac{1}{2} + \frac{\sqrt{3}}{2}i \end{aligned}$$

↓



$$\arg \frac{r-d}{B-d} = \frac{\pi}{3}$$

よす

$$\angle Bdr = \frac{\pi}{3}$$

$$\angle BAC = \frac{\pi}{3} \quad \dots \textcircled{1}$$

また

$$\begin{aligned} \left| \frac{r-d}{B-d} \right| &= \left| \frac{1}{2} + \frac{\sqrt{3}}{2}i \right| \\ &= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1 \end{aligned}$$

$$\left| \frac{r-d}{B-d} \right| = 1$$

$$\frac{|r-d|}{|B-d|} = 1$$

$$\frac{AC}{AB} = 1$$

$$AC = AB \quad \dots \textcircled{2}$$

①, ② よす

$\angle A = \frac{\pi}{3}$, $AB=AC$ の三角形

よす

$\triangle ABC$ は正三角形

$$\therefore \angle A = \angle B = \angle C = \frac{\pi}{3}$$