練22 P30

点 $A(\alpha = c+i)$, $B(\beta = 1)$, $C(\gamma = 3i)$

(1) 3点 A, B, C が一直線上にあるとき

$\dfrac{\gamma-\alpha}{\beta-\alpha}$ が実数となるので

$$\frac{\gamma-\alpha}{\beta-\alpha} = \frac{3i-(c+i)}{1-(c+i)}$$

$$= \frac{-c+2i}{(1-c)-i}$$

$$= \frac{\{-c+2i\}\{(1-c)+i\}}{\{(1-c)-i\}\{(1-c)+i\}}$$

$$= \frac{-c(1-c) - ci + 2(1-c)i + 2i^2}{(1-c)^2 - i^2}$$

$$= \frac{-c+c^2 + \{2(1-c)-c\}i - 2}{1-2c+c^2+1}$$

$$= \frac{c^2-c-2 + (2-3c)i}{c^2-2c+2}$$

$$= \frac{c^2-c-2}{c^2-2c+2} + \frac{2-3c}{c^2-2c+2}i$$

実数となるので

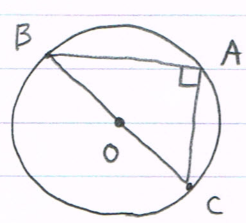
$$\frac{2-3c}{c^2-2c+2} = 0$$

よって

$2-3c = 0$

$c = \dfrac{2}{3}$

(2)



$\angle BAC = 90^\circ$ となるので

$AB \perp AC$ より

$\dfrac{\gamma-\alpha}{\beta-\alpha}$ が純虚数となる

(1)より

$$\frac{\gamma-\alpha}{\beta-\alpha} = \frac{c^2-c-2}{c^2-2c+2} + \frac{2-3c}{c^2-2c+2}i$$

よって

$$\frac{c^2-c-2}{c^2-2c+2} = 0$$

よって

$c^2-c-2=0$

$(c-2)(c+1)=0$

$c = 2, -1$

練23 P30

$2\gamma - (1+\sqrt{3}i)\beta = (1-\sqrt{3}i)\alpha$

$2\gamma - (1+\sqrt{3}i)\beta = -(-1+\sqrt{3}i)\alpha$

$= -(1+\sqrt{3}i-2)\alpha$

$= -(1+\sqrt{3}i)\alpha + 2\alpha$

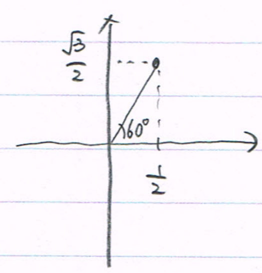
$2\gamma - 2\alpha = (1+\sqrt{3}i)\beta - (1+\sqrt{3}i)\alpha$

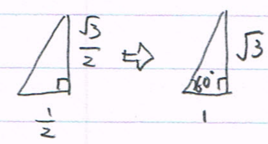
$2(\gamma-\alpha) = (1+\sqrt{3}i)(\beta-\alpha)$

$$\frac{\gamma-\alpha}{\beta-\alpha} = \frac{1+\sqrt{3}i}{2}$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2}i$$

↓





$$\arg\frac{\gamma-\alpha}{\beta-\alpha} = \frac{\pi}{3}$$

よって

$\angle\beta\alpha\gamma = \dfrac{\pi}{3}$

$\angle BAC = \dfrac{\pi}{3}$ …①

また

$$\left|\frac{\gamma-\alpha}{\beta-\alpha}\right| = \left|\frac{1}{2} + \frac{\sqrt{3}}{2}i\right|$$

$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

$$\left|\frac{\gamma-\alpha}{\beta-\alpha}\right| = 1$$

$$\frac{|\gamma-\alpha|}{|\beta-\alpha|} = 1$$

$$\frac{AC}{AB} = 1$$

$AC = AB$ …②

①, ②より

$\angle A = \dfrac{\pi}{3}$, $AB = AC$ の三角形

よって

$\triangle ABC$ は正三角形

$\therefore \angle A = \angle B = \angle C = \dfrac{\pi}{3}$